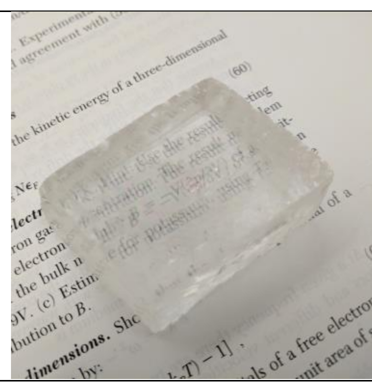
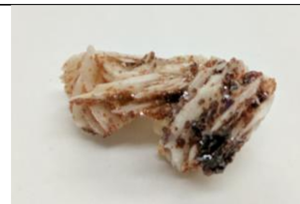
Lecture 1:

- **Presentation about hbar from A. Jurgens**
- **Go over syllabus which is posted on Canvas; Problem sets and course notes will also be posted on canvas, so try to confirm early if you can access canvas**
- Go over prerequisites and check which concepts students have seen already
    - Quantum:
        1. Hamiltonians
        2. Particle in a box
        3. Harmonic oscillator
    - Math
        1. Fourier transform
        2. Vector calculus
        3. Partial differential equations
    - Thermal physics
    - E&M
- Test and homework policy
- **Set office hours**
    - **Options:**
      Thurs 3-4PM
      Friday 2:30-3:30PM
      Monday 11AM-12PM
      Monday 2-3PM (do one or the other)
- **What is solid-state physics?**
    - Study of crystalline solids; **what is a crystal?** (colloquial vs physics) An effectively infinite, regularly repeating structure made up of atoms or molecules
      Demo: geological specimens

	
Quartz Chemical formula: SiO_2, one of the most important chemicals around (also used	Calcite Chemical formula: $CaCO_3$ (calcium carbonate; same

for window glass and as an insulating layer in silicon based electronics) Crystal system: trigonal	chemical as limestone an old-school chalk) Crystal system: trigonal Key property: birefringence—different index of refraction along different crystallographic axes
	
Iron pyrite Chemical formula: FeS_2 Simple cubic	Vanadinite on Barite Vanadinite: $Pb_5(VO_4)_3Cl$, crystal system—hexagonal Barite: $BaSO_4$ (barium sulfate); crystal system=orthorhombic Use to visualize polycrystalline systems

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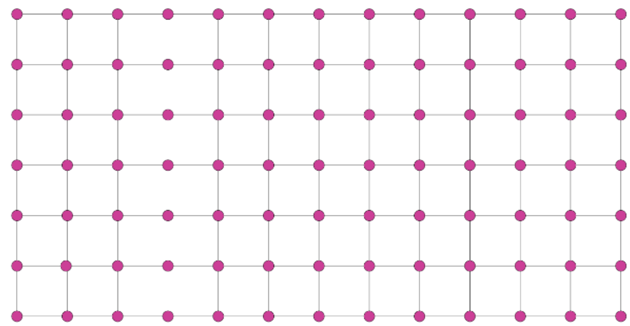
- Applied quantum mechanics, E&M, and thermal/statistical physics to explain mechanical, thermal, and electronic properties of crystalline solids
- This subject relates to many observations in our everyday life (why is window glass transparent? Why does metal conduct electricity?
- Quantum mechanics: there are a handful of problems we can solve exactly (particle encountering rectangular barrier/well; harmonic oscillator; hydrogen atom), and everything else, we need to play tricks like making our difficult problem look close to one of the ones we can solve or using symmetry arguments to make our difficult problem look easier
- Important theme in solid state physics: symmetry
    1. What is symmetry? (colloquial vs physics) An operation under which the system remains invariant. Most symmetries in solid state physics are discrete (finite operations in a set) rather than continuous

- **What operations render an infinite 2D square invariant?**



1. Translation by an integer multiple of a, where a is spacing between lattice points (but continuous translation symmetry is broken)
2. Rotation by pi/2 (but continuous rotation symmetry is broken)
3. Reflection across plane oriented perpendicular to page at 0, pi/4, pi/2, 3pi/4 and equivalent angles
4. Inversion (x,y)→(-x,-y)

- **Go over course structure**
  - Many bits and pieces, but all of it are concepts that contemporary researchers actually use
  - Master Hamiltonian for all of solid state physics (excluding magnetism): consider electrons and nuclei separately
    H=(Kinetic energy of electrons)+(Kinetic energy of nuclei)+(pairwise Coulomb repulsion between negatively charged electrons)+(pairwise Coulomb repulsion between positively charged nuclei)+(Coulomb attraction between nuclei and electrons)

$$\mathcal{H}_{solid} = -\frac{\hbar^2}{2m_e}\sum_{i=1}^{N_e}\vec{\nabla}_i^2 - \frac{\hbar^2}{2m_n}\sum_{\alpha=1}^{N_n}\vec{\nabla}_\alpha^2 + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2}\sum_{\alpha\neq\beta}\frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{R}_\beta|} - \sum_{i,\alpha}\frac{Z_\alpha e^2}{|\vec{r}_i - \vec{R}_\alpha|}$$

Definitions:
m_e=mass of electron
m_n=mass of nuclei
N_e=total number of electrons
N_n=total number of nuclei
$\vec{r}_i, \vec{r}_j$=spatial coordinate of i-th and j-th electron
$\vec{R}_\alpha, \vec{R}_\beta$=spatial coordinates of nuclei

∇_i=spatial derivative with respect to electron coordinate
∇_α=spatial derivative with respect to nuclei coordinates
Z_α=atomic number of alpha-th nucleus

- We can write everything that is happening in a solid, but we cannot solve for the eigenvalues. Thus, we employ known symmetries to simplify the problem, we isolate each piece and try to reach a formulation for its individual contribution, and we make reasonable approximations about what terms can be ignored under what circumstances. Solid state physics, as it is taught in the classroom, is an exposition of these simplification
- The structure of this course is as follows
  1. Ignore electrons and only consider periodic arrangement of nuclei
     - Crystal lattices and crystal structures
       - Math, apply concepts of symmetry
       - A part of the course which doesn't necessarily require any knowledge of physics at all, but one you should cement in your mind as a prerequisite
       - This simplifies this term: $\frac{1}{2}\sum_{\alpha\neq\beta}\frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha-\vec{R}_\beta|}$
     - Measuring crystal structure
       - Reciprocal lattice: take advantage of the fact that a crystal consists of the same group of atoms repeated over and over and express crystal structures in reciprocal space
       - This is one of the most important and widely useful sections of the course, but also the largest conceptual leap
  2. Go back to a bird's eye view and explore why crystals are stable in the first place as well as their aggregate mechanical properties
     - Van der Waals, ionic, metallic, and covalent bonding
     - Elastic strain and elastic waves in solids
  3. Consider vibrations of nuclei around equilibrium positions
     - Phonons: quanta of lattice vibration
     - This chapter focuses on this term: $\frac{\hbar^2}{2m_n}\sum_{\alpha=1}^{N_n}\vec{\nabla}_\alpha^2$
  4. Consider electrons, but ignore atoms
     - Free fermi gas: electrons do not interact with each other except for Pauli exclusion
     - This chapter focuses on this term: $\frac{\hbar^2}{2m_e}\sum_{i=1}^{N_e}\vec{\nabla}_i^2$

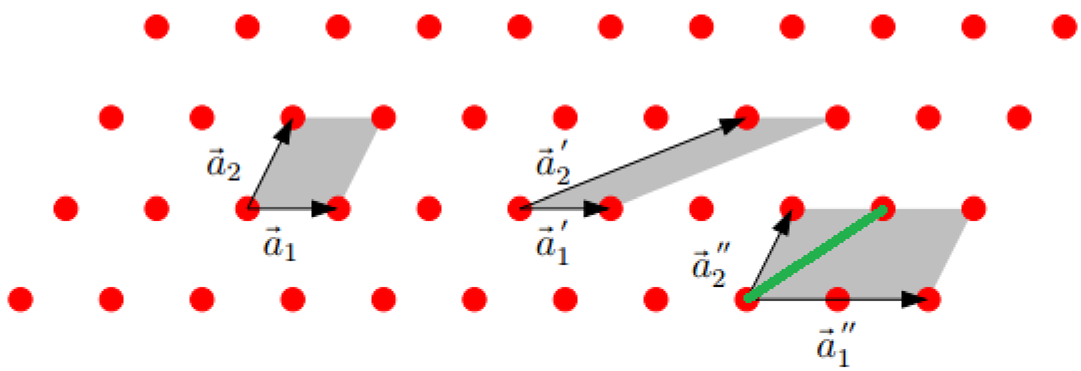
Fundamental crystal lattice systems

Vocabulary:

Lattice: mathematical grid of regularly repeating **points**

Basis: group of atoms attached to these points

Primitive lattice: Set of all lattice points with position vectors $\vec{\boldsymbol{R}} = u_1\vec{\boldsymbol{a}}_1 + u_2\vec{\boldsymbol{a}}_2 + u_3\vec{\boldsymbol{a}}_3$ where $u_{1,2,3}$ are integers and $\vec{\boldsymbol{a}}_1, \vec{\boldsymbol{a}}_2, \vec{\boldsymbol{a}}_3$ are **primitive translation vectors** which can be used to construct a grid with *all* the lattice points. In the image below, $\vec{\boldsymbol{a}}_1, \vec{\boldsymbol{a}}_2$ and $\vec{\boldsymbol{a}}_1', \vec{\boldsymbol{a}}_2'$ are both primitive translation vectors, but $\vec{\boldsymbol{a}}_1'', \vec{\boldsymbol{a}}_2''$ are not because a linear combination of them cannot be used to construct the translation vector shown in green.



Primitive cell: the parallelepiped defined by the D primitive translation vectors in D-dimensional space. In the figure above (representing a lattice in 2 dimensions), the grey boxes bordered by , $\vec{\boldsymbol{a}}_1, \vec{\boldsymbol{a}}_2$ and $\vec{\boldsymbol{a}}_1', \vec{\boldsymbol{a}}_2'$ are two examples of primitive cells

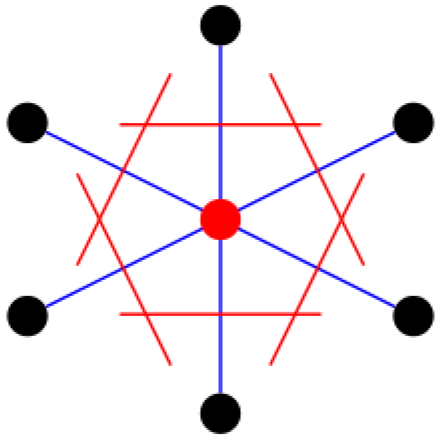
Properties of a primitive cell:

- Only contains one lattice point (depending how the primitive cell is constructed, a given lattice point might be shared by several neighboring cells. For example, the primitive cell defined by $\vec{\boldsymbol{a}}_1, \vec{\boldsymbol{a}}_2$ intersects with 4 lattice points, but each of these lattice points is shared among 4 adjacent cells, giving 4*1/4 =1 point per cell)
- Can be used to tile all of space
- The volume of the primitive cell is the smallest possible volume which can be used to tile all of space and all choices for the primitive cell yield the same volume given by $V_c = |\vec{\boldsymbol{a}}_1 \cdot \vec{\boldsymbol{a}}_2 \times \vec{\boldsymbol{a}}_3|$ in 3 dimension; in 2D $V_c = |\vec{\boldsymbol{a}}_1 \times \vec{\boldsymbol{a}}_2|$

How to construct a primitive cell:

- Guess + inspection
    1. Can this shape I guessed be used to tile the entire lattice?
    2. Does this shape contain only one lattice point?
- The **Wigner-Seitz cell** is a specific type of primitive cell constructed from a lattice given the following procedure:
    1. Pick one lattice point and draw lines to connect this to all nearby lattice points

2. Draw perpendicular bisectors through all of these lines
3. The shape formed from the intersection of the perpendicular bisectors is the Wigner-Seitz cell



The Wigner-Seitz cell will become important in later chapters when we study the reciprocal lattice.

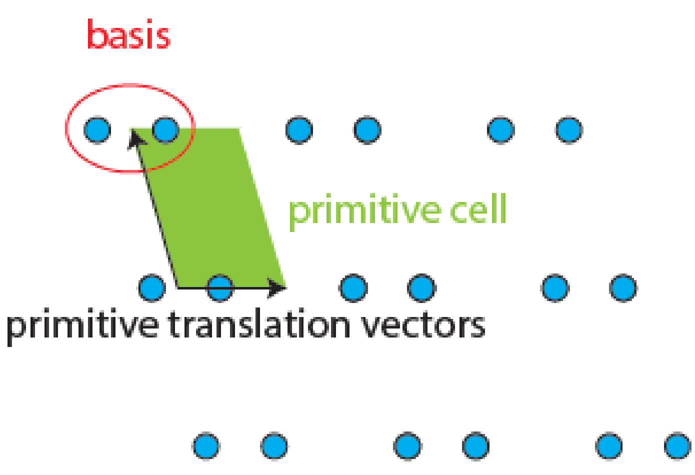
Unit cell: Smallest (or sometimes, most convenient) repeating unit which comprises a crystal. Consists of lattice and basis (will discuss this in 2nd lecture)

Examples:

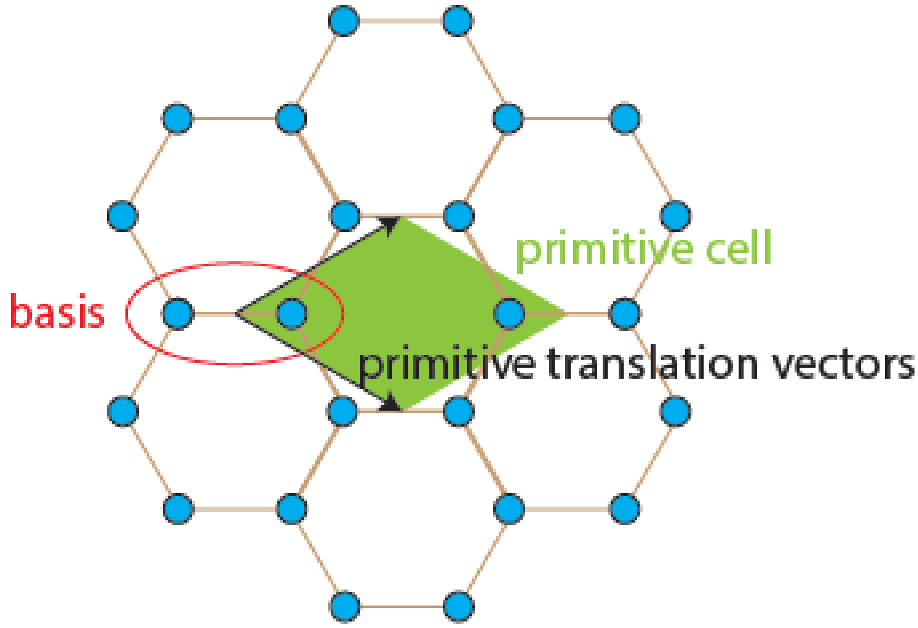
-Are these primitive lattices?

-Find set of lattice points, primitive translation vectors, and basis:

1.



2. Honeycomb lattice



3. Body centered rectangular